Information Manipulation and Social Coordination

— very preliminary —

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Abstract

This paper studies information manipulation in a sender/receiver game with many imperfectly coordinated receivers. An individual receiver wants both to align their choices with an imperfectly observed state and also to align their individual choices with the choices made by fellow receivers. The sender is informed about the state and seeks to *prevent* the receivers coordinating on it. To prevent coordination, the sender takes a costly hidden action that influences the receivers' individual signals. In equilibrium, the sender is unable to introduce any bias into the signals. But the manipulation is nonetheless *always payoff-improving* for the sender. This is because the manipulation endogenously reduces the signal precision, making it harder for the receivers to coordinate. The sender's gains from manipulation are large when the fundamentals of the economy are such that (i) the receivers' signals are intrinsically (technologically) precise, and (ii) the receivers have a strong preference for coordination.

Keywords: coordination, costly talk, strategic uncertainty, bias, noise.

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1 Introduction

How hard is it to persuade a group of rational decision makers to *not coordinate* on an outcome they desire? Can an informed agent with a vested interest in preventing coordination influence the other decision makers' beliefs so as to achieve that aim? Even when their opponents are perfectly rational and understand the persuader's incentives, and so are well-placed to neutralise any attempt to manipulate their beliefs?

This paper addresses these questions in the setting of a sender/receiver game with many imperfectly coordinated receivers. There are a large number of heterogeneously informed receivers who seek to both (i) make choices that are appropriate for their economic conditions, in particular to align their choices with an unobserved state of the world, and (ii) make choices that are similar to those of their fellows, in particular to align their choices with the population average choices of the other information receivers. This latter feature of the receivers' preferences gives a "forecasting-the-forecasts-of-others" or Keynesian "beauty contest" dimension to their problem, as in Morris and Shin (2002). That is, each individual is not only concerned with making accurate predictions about the unobserved state of the world but also making accurate predictions about the choices of their fellows (which may or may not line up with the state of the world). The setup is such that if the state of the world was known with certainty, then these individuals would in fact, in equilibrium, coordinate on their preferred outcome. I then confront these receivers with a single informed sender who seeks to *prevent* them from coordinating. The sender is endowed with an information manipulation technology that allows them to influence the receivers' signals about the state at a cost that is increasing in the size of bias that the sender is attempting to introduce.

To keep the model tractable, and as is standard in the literature, I assume quadratic preferences (so that best response functions are linear) and normal priors and signal distributions. I study equilibria that are linear in the sense that individual strategies are linear functions of their signals. In equilibrium, the strategies and beliefs of the information receivers and the information manipulation of the sender must be mutually consistent.

The main results of the paper are as follows. There is a *unique* (linear) equilibrium with information manipulation. In this equilibrium, individuals receive information that is *unbiased* — i.e., the receivers' *expectations* are not influenced by the sender's manipulation. In equilibrium, the sender is taking costly actions to manipulate information, but these actions are internalized and filtered out by the receivers. But despite this lack of bias, the information manipulation is *always payoff-improving* for the sender — even though the sender's actions are costly to the sender and yet not delivering any change in the receivers' expectations. The sender benefits despite the lack of bias because its manipulation endogenously makes signals *noisier* (and indeed endogenously correlated across receivers) and this extra noise interferes with the receivers' ability to coordinate on the state.

The sender's gain from information manipulation is large when the sender is able to achieve a large reduction in how responsive the receivers' actions are to their individual signals — i.e., if the fundamentals of the economy are such that (i) *in the absence of manipulation* receivers' actions would be highly responsive to their individual signals, but where also (ii) *in the presence of manipulation* individuals are not responsive to their individual signals, then the sender will gain a lot from information manipulation. Roughly speaking, (i) is satisfied when the *intrinsic* precision of the receivers' signals is very high (i.e., when the underlying or "technological" precision of the signals, absent manipulation, is very high). Similarly, (ii) is satisfied if either the intrinsic precision of the receivers' signals is very low or if the receivers have a strong preference for coordination. In short, (i) and (ii) will be jointly satisfied, and hence the sender's gain from information manipulation will be large, when signals are intrinsically precise *and* the receivers have a strong preference for coordination.

Strategic information transmission. This paper is related to the large literature on strategic communication following Crawford and Sobel (1982). As in Kartik, Ottaviani and Squintani (2007) and Kartik (2009), the sender is endowed with a technology for sending biased messages at a cost. The fact that the sender's information manipulation operates in equilibrium not via the receivers' posterior expectations but rather via other features of the posterior distribution, in particular the posterior precision of beliefs, is reminiscent of the Bayesian persuasion literature following Kamenica and Gentzkow (2011). Several features of the model distinguish the setup and are crucial for the results. First, instead of a single information receiver there are many information receivers and moreover they are playing a coordination game with a "forecasting-the-forecasts-of-others" dimension. Second, rather than having their own ideal point or desired outcome, the sender is seeking only to *prevent coordination on the state*, i.e., the sender is not trying to direct coordination to some other point — in this sense the sender is somewhat "agnostic" (or perhaps nihilistic) about outcomes, so long as they inhibit coordination on the state. This also distinguishes the model from my own previous work (Edmond, 2013, 2015) where the sender is seeking to achieve coordination on a particular status quo outcome and so is not agnostic.

Section 2 outlines the model. Section 3 reviews the special case of no manipulation where the model reduces to the setting of Morris and Shin (2002). Section 4 shows that there is a unique linear equilibrium and explains how the sender's manipulation affects the precision of equilibrium beliefs even though it does not introduce bias. Section 5 explains how the information receivers' equilibrium response to their signals depends on the key parameters of the model. Section 6 then explains the sender's gains (and the receivers' losses) from information manipulation.

2 Model

There is a unit mass of ex ante identical information *receivers*, indexed by $i \in [0, 1]$, and a single *sender* attempting to influence their beliefs.

Information receivers' payoffs. The receivers face a *coordination problem* — each would like their action $a_i \in \mathbb{R}$ to be close to the population aggregate action $A := \int_0^1 a_i \, di$, but would also like their action a_i to be close to the true underlying state variable $\theta \in \mathbb{R}$ (about which they are imperfectly informed). In particular, each individual chooses a_i to minimize the expected value of the quadratic loss

$$\lambda(a_i - A)^2 + (1 - \lambda)(a_i - \theta)^2 \tag{1}$$

where the parameter $\lambda \in (0, 1)$ measures the relative strength of the coordination motive. In the special case where θ is known with *certainty*, the optimal action is

$$a_i = \lambda A + (1 - \lambda)\theta \tag{2}$$

Under certainty, in equilibrium each individual takes action $a_i = A = \theta$ and thus in this special case there is no tension between aligning with θ and aligning with the rest of the population — i.e., in this case, the population is perfectly coordinated on θ.

Informed sender and information manipulation. The sender knows the value of θ and seeks to *prevent* the receivers from coordinating on it. In particular, the sender obtains a gross benefit $(\theta - A)^2$ that is increasing in the discrepancy between the aggregate action A and the state θ. The sender is endowed with a technology that, in principle, gives it some influence over the receivers' beliefs. Knowing θ, the sender may take a costly action $b \in \mathbb{R}$ in an attempt to induce the receivers into believing the state is actually $y = \theta + b$, i.e., the term $b = y - \theta$ is a measure of the *bias* that the sender is attempting to introduce. This manipulation incurs a quadratic cost $(y - \theta)^2$. The net payoff to the sender is then

$$V = (\theta - A)^2 - (y - \theta)^2 \tag{3}$$

The receivers know the sender's objective but are imperfectly informed about θ.

Information receivers' signals. Each receiver draws a signal

$$x_i = y + \varepsilon_i = \theta + b + \varepsilon_i \tag{4}$$

where the idiosyncratic noise ε_i is IID normal across receivers, independent of θ, with mean zero and precision $\alpha > 0$ (i.e., variance $1/\alpha$). Information receivers have common priors for θ given by $\theta = z - \eta$ where the common noise η is independent of θ and normally distributed with mean zero and precision $\beta > 0$ (i.e., variance $1/\beta$). In other words, receivers have one source of information, the prior, that is free of the sender's influence and another source of information, the signal x_i, that is not. While the informativeness of the prior is fixed, the informativeness of the signal needs to be determined endogenously in equilibrium.

Before characterizing equilibrium outcomes in the general model with information manipulation, I first review equilibrium outcomes when there is no manipulation. This provides a natural benchmark against which the sender's technology for manipulation can be evaluated.

3 Morris-Shin benchmark

Suppose the sender *cannot* manipulate information — i.e., fix $b = 0$ so that $y = \theta$. Then this model reduces to the setting of Morris and Shin (2002). The optimal action of a receiver with signal x_i and prior mean z satisfies, in general,

$$a(x_i, z) = \lambda \mathbb{E}[\, A(\theta, z) \,|\, x_i, z\,] + (1-\lambda)\mathbb{E}[\,\theta \,|\, x_i, z\,] \tag{5}$$

In this linear-normal setup, each receiver's posterior beliefs are also normal. In particular, with $y = \theta$ a receiver with signal x_i and prior mean z has posterior mean

$$\mathbb{E}[\,\theta \,|\, x_i, z\,] = \frac{\alpha}{\alpha+\beta} x_i + \frac{\beta}{\alpha+\beta} z \tag{6}$$

and all receivers have posterior precision $\alpha + \beta$.

Linear strategies. Following Morris and Shin, I restrict attention to equilibria in which individual receivers use *linear* strategies of the form

$$a(x_i, z) = k x_i + h z \tag{7}$$

for some coefficients k, h to be determined in equilibrium. Since $y = \theta$ the corresponding aggregate action is likewise

$$A(\theta, z) = k\theta + hz \tag{8}$$

Linear equilibrium. Thus, in a linear equilibrium, individual actions satisfy

$$\begin{aligned}
a(x_i, z) &= \lambda \mathbb{E}[\, A(\theta, z) \,|\, x_i, z] + (1-\lambda)\mathbb{E}[\,\theta \,|\, x_i, z] \\
&= \lambda\left(k\mathbb{E}[\,\theta \,|\, x_i, z] + hz\right) + (1-\lambda)\mathbb{E}[\,\theta \,|\, x_i, z] \\
&= (\lambda k + 1 - \lambda)\,\frac{\alpha}{\alpha+\beta} x_i + \left((\lambda k + 1 - \lambda)\,\frac{\beta}{\alpha+\beta} + \lambda h\right) z
\end{aligned} \tag{9}$$

Equating coefficients between (7) and (9) allows us to solve for the equilibrium responses to the signal x_i and prior mean z. In particular, equating the sums of the coefficients

$$k + h = (\lambda k + 1 - \lambda) + \lambda h$$

so that $k + h = 1$. In short, actions simply have the form $a(x_i, z) = kx_i + (1-k)z$. Moreover the coefficient k on the signal x_i must satisfy the fixed-point condition

$$k = (\lambda k + 1 - \lambda)\,\frac{\alpha}{\alpha + \beta} \tag{10}$$

which has the unique solution

$$k^*_{\mathrm{MS}} = \frac{(1-\lambda)\alpha}{(1-\lambda)\alpha + \beta} \in (0, 1) \tag{11}$$

To interpret this formula, first observe that in the absence of the coordination motive ($\lambda \to 0$) this would simply be $\alpha/(\alpha + \beta)$ — i.e., the same weight as given to their signal x_i in the posterior expectation (6) — that depends only on the intrinsic precision of the signal relative to that of the prior, α/β. Put differently, in the absence of the coordination motive, the actions $a(x_i, z) = kx_i + (1 - k)z$ would respond to each piece of information in line with their relative precision. But in the presence of the coordination motive, $\lambda > 0$, the effective weight on the signal x_i gets reduced from α to $(1 - \lambda)\alpha$ with the effective weight on the prior mean z correspondingly increased. Intuitively, in the presence of the coordination motive the common prior gets more weight than its intrinsic precision would warrant. If the coordination motive is very strong ($\lambda \to 1$), each receiver completely ignores their individual signal even if it is intrinsically much more precise than the prior.

With this benchmark in mind, I now return to the general model with manipulation.

4 Equilibrium with information manipulation

Now suppose the sender *is* able to manipulate information. In this setting the equilibrium fixed point problem is more complicated because the receivers' beliefs and the sender's incentives to manipulate information need to be mutually consistent.

As in Morris and Shin, I again restrict attention to linear strategies of the form

$$a(x_i, z) = kx_i + hz \tag{12}$$

for some coefficients k, h to be determined. Hence as a function of the sender's y the corresponding aggregate action is now

$$A(y, z) = ky + hz \tag{13}$$

Sender's manipulation problem. Given an aggregate action of this form, the sender's problem is to choose $y \in \mathbb{R}$ to maximize

$$V(y) = (\theta - ky - hz)^2 - (y - \theta)^2$$

The solution to the sender's problem is

$$y(\theta, z) = \frac{1}{1+k}\theta + \frac{kh}{(1+k)(1-k)}z \tag{14}$$

Given $y(\theta, z)$, the aggregate action will be

$$A(y(\theta, z), z) = ky(\theta, z) + hz = \frac{k}{1+k}\theta + \frac{h}{(1+k)(1-k)}z \tag{15}$$

The second-order condition for the sender's problem requires $1 - k^2 > 0$. Hence in what follows I will restrict attention to $k \in (-1, +1)$.

Information receivers' actions. The optimal action of an individual receiver is again given by (5), above, so that in any linear equilibrium

$$\begin{aligned} a(x_i, z) &= \lambda \left(\frac{k}{1+k}\mathbb{E}[\,\theta \,|\, x_i, z\,] + \frac{h}{(1+k)(1-k)}z \right) + (1-\lambda)\mathbb{E}[\,\theta \,|\, x_i, z\,] \\ &= \left(1 - \frac{\lambda}{1+k}\right)\mathbb{E}[\,\theta \,|\, x_i, z\,] + \frac{\lambda}{1+k}\frac{h}{1-k}z \end{aligned} \tag{16}$$

Thus the main task is to calculate the conditional expectation $\mathbb{E}[\,\theta \,|\, x_i, z\,]$ when the sender engages in manipulation. Once that conditional expectation is in hand, the equilibrium problem can be solved by matching coefficients as before.

Information receivers' signal-extraction problem. Given the sender's policy (14), receivers have endogenous signals of the form

$$x_i = y(\theta, z) + \varepsilon_i = \frac{1}{1+k}\theta + \frac{kh}{(1+k)(1-k)}z + \varepsilon_i$$

And since the prior is $z = \theta + \eta$ this is equivalently

$$x_i = \frac{1-k+kh}{(1-k)(1+k)}\theta + \frac{kh}{(1+k)(1-k)}\eta + \varepsilon_i \tag{17}$$

The implications of this form of signal become transparent when we observe that:

LEMMA 1. In any linear equilibrium the coefficients k, h sum to $k + h = 1$.

Unbiased signals. Plugging $h = 1 - k$ into (17) and simplifying then gives

$$x_i = \theta + \frac{k}{1+k}\eta + \varepsilon_i \tag{18}$$

This implies:

Proposition 1. In any linear equilibrium the endogenous signals x_i are unbiased for θ. The endogenous noise

$$\xi_i := \frac{k}{1+k}\eta + \varepsilon_i \tag{19}$$

has mean zero and precision $\gamma(k) \leq \alpha$ given by

$$\gamma(k) = \frac{\alpha\beta}{\alpha\delta(k) + \beta}, \qquad \delta(k) := \left(\frac{k}{1+k}\right)^2 \geq 0 \tag{20}$$

In short, the endogenous signals are unbiased for θ but are more noisy than their intrinsic precision warrants, $\gamma(k) \leq \alpha$.

Sender's manipulation. Although the private signals x_i are unbiased for θ in equilibrium, this *does not* mean that the sender is not manipulating information. In any linear equilibrium, the sender chooses

$$y(\theta, z) = \frac{1}{1+k}\theta + \frac{k}{1+k}z$$

and hence is taking the action

$$b(\theta, z) = y(\theta, z) - \theta = \frac{k}{1+k}(z - \theta) = \frac{k}{1+k}\eta \tag{21}$$

and thus, in general, the sender is indeed actively manipulating information in a state-contingent manner — it is just that the receivers understand and internalize the sender's incentives so that their individual signals remain unbiased in equilibrium.

Aggregate action. With $A(y, z) = ky + (1 - k)z$ and $y(\theta, z) = (\theta + kz)/(1 + k)$ we then have that in any linear equilibrium the aggregate action is

$$A(y(\theta, z), z) = \theta + \frac{1}{1+k}(z - \theta) = \theta + \frac{1}{1+k}\eta \tag{22}$$

As in the Morris-Shin benchmark, the common prior z acts as a form of "aggregate noise" that in general impedes the receivers' from coordinating on the true state θ. Notice that for given $z - \theta$ the magnitude of the discrepancy between the aggregate action and θ will be greater the smaller is the response coefficient k. Put differently, the more receivers respond to z, the larger will be the variation of the aggregate action around θ. In the knife-edge case that the true θ coincides with z, the sender does not actively manipulate information ($b = 0$, so $y = \theta$) and the aggregate action hits θ exactly.

Endogenous noise and cross-sectional correlation. While the sender's manipulation does not lead to any bias in x_i in equilibrium, it generally makes the endogenous private signals x_i both *less precise* and more *correlated* in the cross-section of information receivers. In the Morris-Shin benchmark the exogenous idiosyncratic noise ε_i has precision α and is IID across receivers (i.e., the signals x_i are conditionally independent). But with the sender's manipulation, the endogenous noise ξ_i is no longer purely idiosyncratic, it also contains the common term $b(\theta, z)$ from the sender's action, and this both reduces the precision, from α to $\gamma(k)$, and makes the noise ξ_i correlated in the cross-section of receivers (i.e., the signals x_i are no longer conditionally independent). More specifically, conditional on θ the correlation between any two individuals i and j is given by

$$\mathrm{Corr}[x_i, x_j \,|\, \theta] = \mathrm{Corr}[\xi_i, \xi_j] = \frac{\alpha\delta(k)}{\alpha\delta(k) + \beta} =: \rho(k) \in [0, 1) \tag{23}$$

where $\delta(k) \geq 0$ as in (20) above.

Notice that if $k = 0$, receivers do not react to their signals x_i and the sender finds it optimal to not engage in any costly manipulation — i.e., $b = 0$ so that $y = \theta$. Hence in this situation the signals x_i do not depend on η, the correlation $\rho(k)$ is zero, and the precision $\gamma(k) = \alpha$. But this will generally not be an equilibrium — because if the sender does not manipulate, then information receivers will want to give their signals at least some consideration and react to them at least a little, $k > 0$, and hence the sender will in turn *not* refrain from manipulation and the receivers' signals will be at least somewhat less precise than α.

We now need to go on and solve for k in equilibrium.

Linear equilibrium. To solve the equilibrium problem, observe that each receiver has two pieces of information, both unbiased for θ: (i) x_i with endogenous precision $\gamma(k)$, and (ii) z with precision β. Since both are normally distributed, the posterior expectation of θ is just

$$\mathbb{E}[\,\theta \,|\, x_i, z\,] = \frac{\gamma(k)}{\gamma(k) + \beta} x_i + \frac{\beta}{\gamma(k) + \beta} z \tag{24}$$

where $\gamma(k)$ is as defined in (20) above. Likewise, the posterior precision of θ is

$$1/\mathrm{Var}[\,\theta \,|\, x_i, z\,] = \gamma(k) + \beta \tag{25}$$

(i.e., less than the posterior precision $\alpha + \beta$ in the Morris-Shin benchmark). Now plugging the expression (24) into (16) and using $k + h = 1$ gives

$$\begin{aligned} a(x_i, z) &= \left(1 - \frac{\lambda}{1+k}\right) \mathbb{E}[\,\theta \,|\, x_i, z\,] + \frac{\lambda}{1+k} z \\ &= \left(1 - \frac{\lambda}{1+k}\right) \frac{\gamma(k)}{\gamma(k) + \beta} x_i + \left(1 - \frac{\lambda}{1+k}\right) \left(\frac{\beta}{\gamma(k) + \beta} + \frac{\lambda}{1+k}\right) z \end{aligned} \tag{26}$$

the coefficients of which indeed sum to one. Thus in a linear equilibrium the coefficient k on the signals x_i satisfies the fixed point condition

$$k = \left(1 - \frac{\lambda}{1+k}\right) \frac{\gamma(k)}{\gamma(k)+\beta}$$

To simplify the characterization of solutions to this problem, it's helpful to write this as

$$L(k) = R(k) \tag{27}$$

where

$$L(k) := k\frac{\gamma(k)+\beta}{\gamma(k)} = k\left(\frac{\alpha+\beta}{\alpha} + \delta(k)\right) \tag{28}$$

and

$$R(k) := 1 - \frac{\lambda}{1+k} \tag{29}$$

All linear equilibria are solutions of the fixed point problem (27). In fact:

Proposition 2.

(i) The fixed point problem $L(k) = R(k)$ has a unique solution $k^* \in (0,1)$. Hence there is a unique linear equilibrium.

(ii) In equilibrium, individual receivers respond less to their signals than in the benchmark with no information manipulation, $k^* < k^*_{\text{MS}}$.

To see the first part of this, observe that $R'(k) > 0$ and $R''(k) < 0$ with $R(0) = 1-\lambda > 0$ and $R(1) = 1 - \lambda/2 < 1$ while $L'(k) > 1$ with $L(0) = 0$ and $L(1) > 1$. Hence by the intermediate value theorem there is a unique $k^* \in (0,1)$ that solves $L(k) = R(k)$. This leaves open the possibility that there is another, negative, solution. Any such solution must be less than $-\sqrt{1-\lambda}$, since until that point it is clearly the case that $R(k) > k > L(k)$. The tricky issue is whether the function $R(k)$ can catch up (and pass) the function $L(k)$ as $k \to -1$. While $R(k)$ and $L(k)$ both diverge to $-\infty$ as $k \to -1$, a l'Hôpital's rule calculation shows that $R(k)/L(k) \to 0^+$ so that $R(k)$ is always greater than $L(k)$. Thus there is in fact no negative solution.[1] Figure 1 illustrates.

[1]Alternatively, the fixed point problem $L(k) = R(k)$ can be rewritten as a cubic equation in k. And one can show that the discriminant of this cubic is negative, which implies that the cubic has one real root and a pair of complex roots. Only the real root is a candidate for an equilibrium. The argument in the text then establishes that the real root is in $(0,1)$.

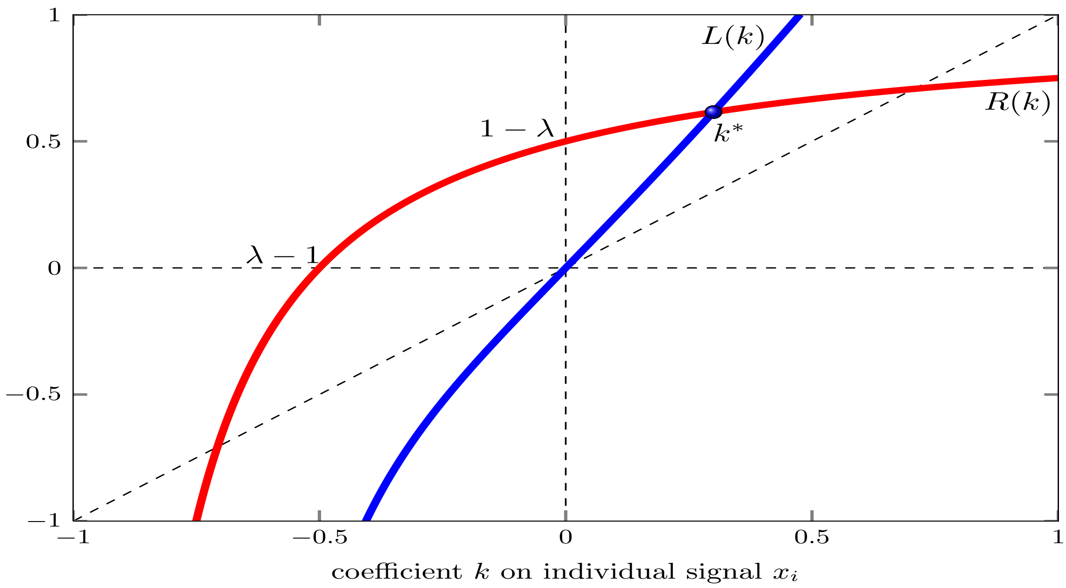


Figure 1: Constructing the unique linear equilibrium.

In equilibrium, the coefficient k on individual signals x_i must lie at the intersection of the red $R(k)$ and blue $L(k)$ curves. An application of the intermediate value theorem shows that there is a positive solution k^*. Both $R(k)$ and $L(k)$ diverge to $-\infty$ as $k \to -1$, but an application of l'Hôpital's rule shows that $R(k) > L(k)$ for all $k < 0$, so there is no negative solution.

Moreover the equilibrium k^* is strictly less than its counterpart Morris-Shin benchmark value k^*_{MS}. Intuitively, this is because with manipulation, each receiver endogenously gives less weight to their signal x_i when forming their posterior expectation $\mathbb{E}[\,\theta\,|\,x_i, z\,]$ and since it is common knowledge that each individual gives less weight to x_i in their beliefs about θ, in equilibrium each individual's actions also respond less to that information.

To summarize, there is a unique coefficient k^* that renders the beliefs of the information receivers and the manipulation of the sender mutually consistent. In this equilibrium, individuals take actions $a(x_i, z) = k^* x_i + (1 - k^*)z$, that respond less to their signal x_i than in the Morris-Shin benchmark without information manipulation, $k^* < k^*_{\text{MS}}$. The precision of the signals is $\gamma(k^*) < \alpha$ with cross-sectional correlation $\rho(k^*) \in (0, 1)$ — i.e., likewise the receivers' signals are both less precise and more correlated across individuals than in the Morris-Shin benchmark.

So we know that receivers are less responsive to their signals, $k^* < k^*_{\text{MS}}$. But *how much* less responsive are they? To figure this out, we need to know how the equilibrium response k^* varies with the underlying parameters of the economy.

5 Comparative statics of the response coefficient k^*

The equilibrium response k^* to an individual signal x_i is a function of two parameters: (i) the *intrinsic* (underlying, technological) precision of the signal relative to the prior, α/β, and (ii) the receivers' preference for coordination, λ. Observe that only the ratio α/β matters for the equilibrium, not the levels themselves. To simplify calculations, let $\omega := \alpha/(\alpha+\beta)$ denote the intrinsic relative precision of the signal, i.e., the weight that an individual's posterior for θ would give to their signal x_i absent manipulation.

The fixed point problem (27) can then be written in the form $L(k\,;\omega) = R(k\,;\,\lambda)$ and an application of the implicit function theorem gives:

Proposition 3. The equilibrium response k^* to an individual signal x_i is:

(i) Strictly increasing in ω for all $\lambda \in (0,1)$ with $\lim_{\omega\to 0} k^* = 0$ and $\lim_{\omega\to 1} k^* =: \underline{k}(\lambda) < \sqrt{1-\lambda} < 1$.

(ii) Strictly decreasing in λ for all $\omega \in (0,1)$ with $\lim_{\lambda\to 0} k^* =: \overline{k}(\omega) < \omega < 1$ and $\lim_{\lambda\to 1} k^* = 0$.

Intuitively, as the intrinsic relative precision ω rises, receivers indeed respond more to their individual signal x_i, and, as the coordination motive becomes stronger, receivers respond more to their common prior z. These effects are *qualitatively* exactly the same as in the Morris-Shin benchmark where the sender cannot manipulate information. But, importantly, the sender's ability to manipulate information *bounds* the willingness of the receivers to respond to their individual signals. To see this effect, observe that in the Morris-Shin benchmark the coefficient on x_i, from (11), can be written

$$k^*_{\text{MS}} = \frac{(1-\lambda)\omega}{(1-\lambda)\omega + (1-\omega)} \in (0,1) \tag{30}$$

hence

$$\lim_{\omega\to 1} k^*_{\text{MS}} = 1, \qquad \text{and} \qquad \lim_{\lambda\to 0} k^*_{\text{MS}} = \omega \tag{31}$$

From part (i) of Proposition 3 we have that as the intrinsic relative precision becomes arbitrarily high, $\omega \to 1$, the equilibrium coefficient $k^* \to \underline{k}(\lambda) < \sqrt{1-\lambda} < 1$, i.e., less that the counterpart limit in the Morris-Shin benchmark. In other words, in the benchmark setting without manipulation $\omega \to 1$ implies receivers respond only to their signals whereas with manipulation the response to x_i is bounded below $\sqrt{1-\lambda} < 1$ precisely because the sender's manipulation makes the signals less precise. For example, if $\omega \to 1$ and $\lambda = 0.75$ then we know $k^* < \sqrt{1-0.75} = 0.5$ with manipulation but $k^*_{\text{MS}} = 1$ absent manipulation, so in this case the gap between k^* and k^*_{MS} is at least 0.5. Moreover, the bound $\sqrt{1-\lambda}$ is *tighter* the more important is the coordination motive. For example, if $\omega \to 1$ but $\lambda = 0.99$ then we know $k^* < \sqrt{1-0.99} = 0.1$ with manipulation but $k^*_{\text{MS}} = 1$ absent manipulation, so in this case the gap between k^* and k^*_{MS} is at least 0.9.

Similarly, from part (ii) of Proposition 3 we have that as the coordination motive fades in importance, $\lambda \to 0$, the equilibrium coefficient $k^* \to \overline{k}(\omega) < \omega < 1$, i.e., less than the counterpart limit in the Morris-Shin benchmark. In other words, in the benchmark setting without manipulation $\lambda \to 0$ implies receivers respond to their signals in accordance with their relative precision ω whereas with manipulation the response is bounded below ω, again because the signals are endogenously less precise.

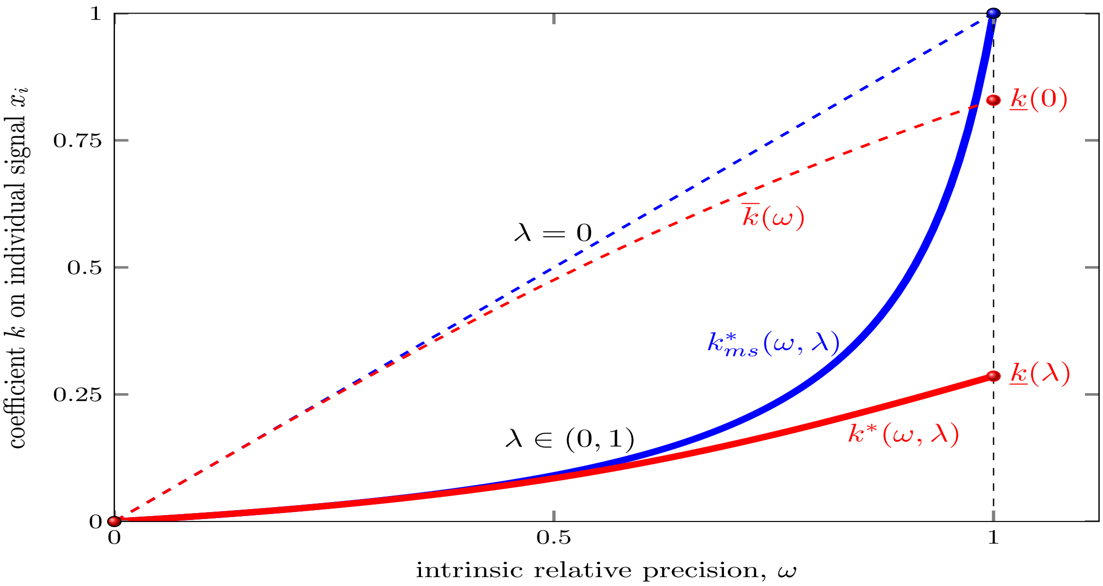


Figure 2: Equilibrium response $k^*(\omega, \lambda)$ to individual signal x_i.

An individual's equilibrium response $k^*(\omega, \lambda)$ to their signal x_i is strictly increasing in the intrinsic relative precision ω and strictly decreasing in the preference for coordination λ. Moreover, individuals always respond less to their signal than they would in the absence of manipulation, $k^*(\omega, \lambda) < k^*_{\text{MS}}(\omega, \lambda)$ for all $\omega \in (0, 1)$ and $\lambda \in (0, 1)$. In particular, the limits $\overline{k}(\omega) = k^*(\omega, 0) < k^*_{\text{MS}}(\omega, 0) = \omega$ and $\underline{k}(\lambda) = k^*(1, \lambda) < k^*_{\text{MS}}(1, \lambda) = 1$.

These comparative statics are illustrated in Figure 2, which shows both k^* (in red) and k^*_{MS} (in blue) as functions of ω for $\lambda = 0$ and some $\lambda \in (0, 1)$. Consider first the case of some $\lambda \in (0, 1)$. We see that both k^* and k^*_{MS} are increasing in ω and that as $\omega \to 1$ we have $k^*_{\text{MS}} \to 1$ but $k^* \to \underline{k}(\lambda) < 1$ so that when ω is high an individual responds distinctly less to their signal than they would in the absence of information manipulation. By contrast, when $\omega \to 0$ we have both $k^*_{\text{MS}} \to 0$ and $k^* \to 0$ so that when ω is low an individual naturally does not respond to their signal at all. In other words, it is in high ω situations where the manipulation is likely to be substantially affecting individual behavior. Now consider the case of no coordination motive, $\lambda \to 0$, so that in the absence of manipulation we would simply have individuals responding in line with the intrinsic relative precision of their signals, $k^*_{\text{MS}} = \omega$ (i.e., the 45°-line in Figure 2). The curve labelled $\overline{k}(\omega)$ shows the limit of no coordination motive when there *is* manipulation, $\lim_{\lambda \to 0} k^* =: \overline{k}(\omega)$, again always less than the Morris-Shin counterpart.

To summarize, in circumstances where receivers are fundamentally inclined to *not* respond to their signals, e.g., when ω is very low, the equilibrium coefficient k^* will itself be both low and fairly close to the Morris-Shin benchmark value k^*_{MS}. After all, if the signal is intrinsically uninformative relative to the prior, the sender's manipulation does not have much to work with. But in circumstances where receivers *are* fundamentally inclined to respond to their signals, e.g., when ω is very high, the equilibrium coefficient k^* can be substantially less than the Morris-Shin benchmark, especially when λ is high.

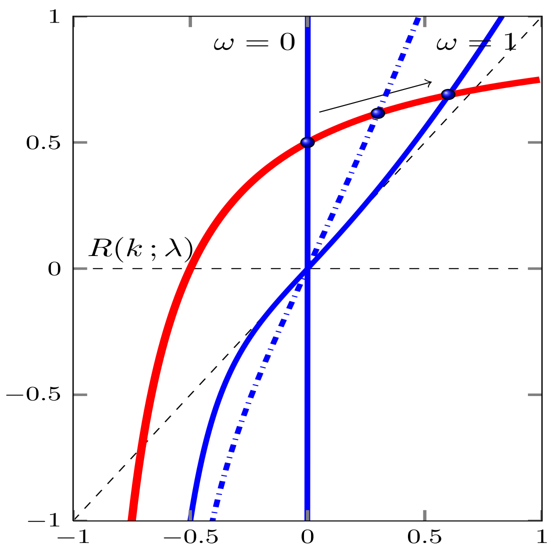
Graphical derivation. To derive the comparative statics graphically, observe that ω only matters for the $L(k\,;\,\omega)$ schedule. As shown in the left panel of Figure 3, as ω increases, the $L(k\,;\,\omega)$ schedule rotates clockwise while the $R(k\,;\,\lambda)$ schedule is unchanged. Hence k^* is strictly increasing in ω. At the extreme of $\omega = 0$, the individual signals are uninformative (regardless of manipulation) and so individuals do not respond to them, $k^* = 0$. At the other extreme of $\omega = 1$, the limiting coefficient $\underline{k}(\lambda)$ solves

$$k(1+\delta(k)) = R(k\,;\,\lambda)$$

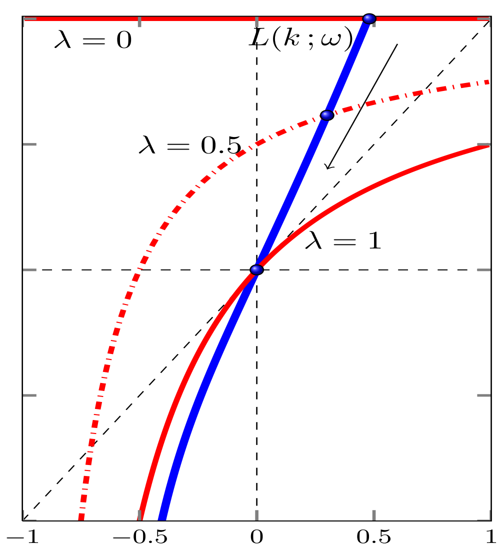
and so, as illustrated in the left panel, this limiting coefficient $\underline{k}(\lambda)$ is strictly less than $\sqrt{1-\lambda} < 1$. Similarly as shown in the right panel of Figure 3, as λ increases, the $R(k\,;\,\lambda)$ schedule shifts down while the $L(k\,;\,\omega)$ schedule is unchanged. Hence k^* is strictly decreasing in λ. Since $R(k\,;\,0) = 1$ for all k the limiting coefficient $\overline{k}(\omega)$ solves

$$k\left(\frac{1}{\omega} + \delta(k)\right) = 1$$

and, since $\delta(k) > 0$ for any $k \in (0,1)$, the solution $\overline{k}(\omega)$ must be strictly less than ω. At the other extreme of $\lambda = 1$, we have $R(k\,;\,1) = k/(1+k)$ which intersects $L(k\,;\,\omega)$ at the origin.



(a) relative precision ω shifts $L(k\,;\,\omega)$



(b) coordination λ shifts $R(k\,;\,\lambda)$

Figure 3: Deriving the comparative statics of $k^*(\omega, \lambda)$

Panel (a) shows that as the intrinsic relative precision $\omega := \alpha/(\alpha+\beta)$ increases, the $L(k\,;\,\omega)$ schedule rotates clockwise while the $R(k\,;\,\lambda)$ schedule is unchanged. Hence $k^*(\omega, \lambda)$ is increasing in ω. Panel (b) shows that as the coordination motive λ increases, the $R(k\,;\,\lambda)$ schedule shifts down while the $L(k\,;\,\omega)$ schedule is unchanged. Hence $k^*(\omega, \lambda)$ is decreasing in λ.

Comparative statics of equilibrium beliefs. With the comparative statics of k^* in hand, we can now work out the implications for equilibrium beliefs. To this end, let $\rho(k\,;\,\omega)$ denote the cross-sectional correlation of receiver signals for given k and intrinsic relative precision ω and let $w(k\,;\,\omega)$ likewise denote the *weight* on the receiver signal x_i in their posterior expectation $\mathbb{E}[\,\theta\,|\,x_i, z]$. From (23) and (24), these can be written

$$\rho(k\,;\,\omega) := \frac{\omega\delta(k)}{\omega\delta(k) + 1 - \omega} \tag{32}$$

and

$$w(k\,;\,\omega) := \frac{\omega}{\omega\delta(k) + 1} \tag{33}$$

and then let $\rho^*(\omega, \lambda) := \rho(k^*(\omega, \lambda);\, \omega)$ and $w^*(\omega, \lambda) := w(k^*(\omega, \lambda);\, \omega)$ denote the equilibrium correlation and equilibrium weight[2] on signals as functions of the underlying parameters ω, λ. In this notation, a receiver's equilibrium posterior expectation is

$$\mathbb{E}[\,\theta\,|\,x_i, z] = w^*(\omega, \lambda)x_i + (1 - w^*(\omega, \lambda))z$$

Some straightforward calculations then give:

PROPOSITION 4.

(i) The correlation ρ^* is (a) strictly increasing in ω with $\lim_{\omega\to 0} \rho^* = 0$ and $\lim_{\omega\to 1} \rho^* = 1$ for all $\lambda \in (0, 1)$, and (b) strictly decreasing in λ with $\lim_{\lambda\to 0} \rho^* =: \overline{\rho}(\omega) > 0$ and $\lim_{\lambda\to 1} \rho^* = 0$ for all $\omega \in (0, 1)$.

(ii) The weight w^* is (a) strictly increasing in ω with $\lim_{\omega\to 0} w^* = 0$ and $\lim_{\omega\to 1} w^* =: \underline{w}(\lambda) < 1$ for all $\lambda \in (0, 1)$, and (b) strictly increasing in λ with $\lim_{\lambda\to 0} w^* =: \overline{w}(\omega) < \omega$ and $\lim_{\lambda\to 1} w^* = \omega$ for all $\omega \in (0, 1)$.

These effects are illustrated in Figure 4. The left panel shows the equilibrium correlation ρ^* as a function of the intrinsic relative precision ω for various levels of the preference for coordination λ. For given λ, the equilibrium correlation is strictly increasing in ω reaching the case of perfectly correlated signals as $\omega \to 1$. Intuitively, as signals become intrinsically precise, receivers respond more to them, k^* is higher, and hence via the common term $b = k^*/(1+k^*)\eta$ introduced by the sender's manipulation, their signals are more correlated (even conditional on θ). Similarly, a higher λ reduces k^* so that signals are both endogenously more precise (driving the precision up towards its benchmark level α) and less correlated across individuals. By contrast, in the Morris-Shin benchmark, conditional on the true θ, signals would be uncorrelated for all ω, λ.

[2] I focus on the equilibrium weight $w^*(\omega, \lambda)$ rather than, say, the equilibrium posterior precision $\gamma(k^*(\omega, \lambda)) + \beta$, because the latter depends on both the relative intrinsic precision α/β and total intrinsic precision $\alpha + \beta$. The weight $w^*(\omega, \lambda)$ is the natural counterpart of the parameter ω (both are scale-free) and comparing the two allows us to see the differences between the intrinsic informativeness of signals and their equilibrium informativeness on a like-for-like basis.

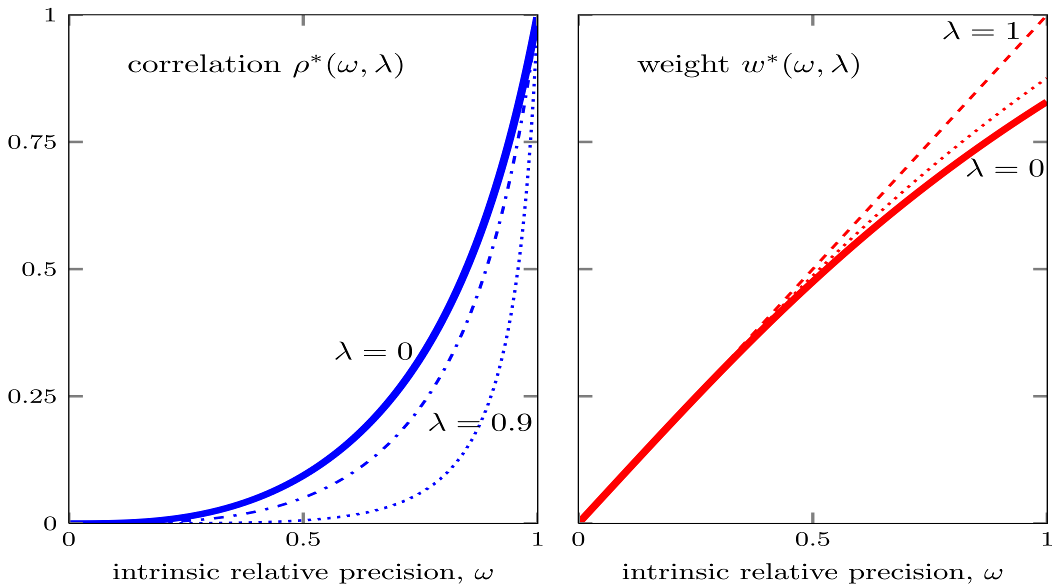


Figure 4: Equilibrium correlation $\rho^*(\omega, \lambda)$ and weight on signals $w^*(\omega, \lambda)$.

The left panel shows the equilibrium (conditional) correlation of signals $\rho^*(\omega, \lambda)$ as a function of the intrinsic relative precision ω and the preference for coordination λ. The correlation is larger when the coefficient k^* is larger and hence the correlation is increasing in ω and decreasing in λ. The right panel shows the equilibrium weight $w^*(\omega, \lambda)$ given to signals in the receivers' posterior expectations, a scale-free measure of the equilibrium relative precision of signals. In the Morris-Shin benchmark this would simply be the intrinsic value ω (i.e., the 45°-line). With manipulation, the equilibrium weight is always less than ω, is increasing in ω and increasing in λ so that the gap between $w^*(\omega, \lambda)$ and its intrinsic value ω is decreasing in λ.

The right panel of Figure 4 likewise shows the equilibrium weight w^* as a function of ω for various levels of λ. Given the sender's manipulation, w^* is always less than its intrinsic value ω (i.e., the value it would take in the Morris-Shin benchmark). The equilibrium weight is strictly increasing in ω and the gap between the equilibrium value w^* and its intrinsic value ω is decreasing in the preference for coordination λ. When $\lambda = 0$, this gap is at its largest, falling as λ rises until in the limit as $\lambda \to 1$ we have $w^* = \omega$ as in the Morris-Shin benchmark (i.e., coinciding with the 45°-line in the right panel). Intuitively, as λ rises individuals are less responsive to their signals (i.e., k^* falls), and so there is again less contamination of signals by the common term $b = (k^*/(1 + k^*))\eta$ and hence the equilibrium signal precision is closer to its intrinsic value.

6 Effectiveness of information manipulation

So the sender can manipulate information in equilibrium. But is this manipulation *effective* in the sense of improving the sender's payoff? It could be that the sender is "trapped" into engaging in costly manipulation that is payoff-reducing simply because the sender cannot credibly commit to not use this option. It turns out that this is not the case. In fact, the sender is made strictly better off by the availability of its technology for manipulating information — *despite the fact that it cannot in equilibrium achieve any bias in signals.*

Sender benefits from information manipulation. The sender's payoffs are

$$V = (\theta - A(y(\theta, z), z))^2 - (y(\theta, z) - \theta)^2 \tag{34}$$

In the Morris-Shin benchmark where the sender cannot manipulate information we have $y = \theta$ and since $A(\theta, z) = k\theta + (1-k)z$ we can collect terms and simplify to obtain

$$V_{\text{MS}}(k\,; \theta, z) := (1-k)^2(\theta - z)^2 \tag{35}$$

The greater is k, the lower are the sender's payoffs. This is because the greater is k, the more the aggregate action becomes tightly distributed around the true θ — in the limit as $k \to 1$, the aggregate action hits θ with certainty — and since the sender prefers that the receivers *do not coordinate* on θ, this reduces the sender's payoffs.

Similarly, *with* information manipulation we have $A(y, z) = ky + (1-k)z$ and now $y(\theta, z) = (\theta + kz)/(1+k)$ so that again collecting terms and simplifying from (34) we get the payoffs

$$V(k\,; \theta, z) := \left(\frac{1-k}{1+k}\right)(\theta - z)^2 \tag{36}$$

Thus as in the Morris-Shin benchmark, the greater is k, the lower are the sender's payoffs. Since the state-contingent term $(\theta - z)^2$ is common across the two scenarios, we can compare the sender's payoffs by focussing on the multiplicative terms $v_{\text{MS}}(k) := (1-k)^2$ for the Morris-Shin case and $v(k) := (1-k)/(1+k)$ for the information manipulation case.[3]

Now let $V^*_{\text{MS}}(\omega, \lambda)$ denote the sender's equilibrium payoffs without information manipulation, namely $v_{\text{MS}}(k)$ evaluated at the benchmark $k = k^*_{\text{MS}}(\omega, \lambda)$ from (11), and likewise let $V^*(\omega, \lambda)$ denote the sender's equilibrium payoffs with information manipulation, namely $v(k)$ evaluated at the $k = k^*(\omega, \lambda)$ implied by (27). We then have:

Proposition 5.

(i) Information manipulation is always payoff-increasing for the sender, $V^* > V^*_{\text{MS}}$.

(ii) Both V^* and V^*_{MS} are (a) strictly decreasing in ω with $\lim_{\omega\to 0} V^*_{\text{MS}} = \lim_{\omega\to 0} V^* = 1$ and $\lim_{\omega\to 1} V^*_{\text{MS}} = 0$ but $\lim_{\omega\to 1} V^* =: \underline{V}^*(\lambda) > 0$ for all $\lambda \in (0,1)$, and (b) strictly increasing in λ for all $\omega \in (0,1)$.

(iii) The sender's **gain from manipulation** $V^* - V^*_{\text{MS}}$ is (a) single-peaked in ω, strictly increasing in ω for all $\omega < \omega^*(\lambda)$ and strictly decreasing in ω for all $\omega > \omega^*(\lambda)$, with (b) critical point $\omega^*(\lambda)$ strictly increasing in λ and satisfying $\omega^*(0) > 0$ and $\omega^*(1) = 1$.

[3] If $z = \theta$ the (ex post) state-contingent payoff of the sender is zero in both cases. In fact, as the policy $b = (k/(1+k))(z - \theta)$ from (21) makes clear the sender only actively manipulates information if $z \neq \theta$. If $z = \theta$ then $b = 0$ and $y = \theta$. Of course the receivers do not know θ and so do not know if $z = \theta$ and hence even in this knife-edge state of the world we have $k^* \in (0, k^*_{\text{MS}})$, signals are endogenously noisier etc.

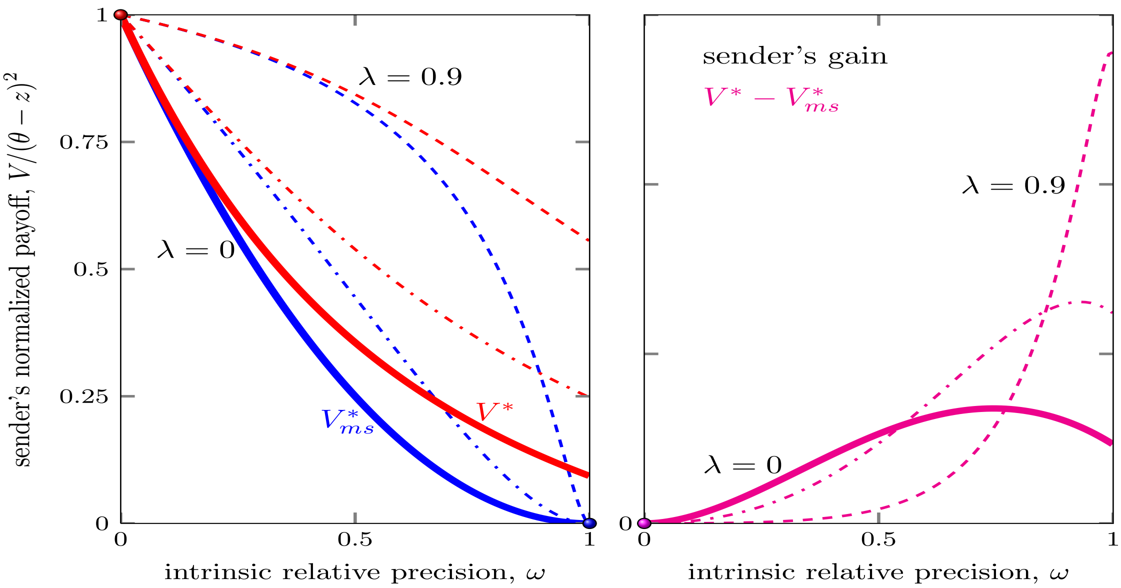


Figure 5: Sender benefits from information manipulation.

Information manipulation is strictly payoff-increasing for the sender, $V^*(\omega, \lambda) > V^*_{\text{MS}}(\omega, \lambda)$. Since the payoffs are both decreasing in the coefficient k on individual signals, from Proposition 3 the sender's payoff is likewise decreasing in the intrinsic relative precision ω but increasing in the preference for coordination λ. The gain from information manipulation $V^*(\omega, \lambda) - V^*_{\text{MS}}(\omega, \lambda)$ is single-peaked in ω, increasing for $\omega < \omega^*(\lambda)$ then decreasing for $\omega > \omega^*(\lambda)$ with critical point $\omega^*(\lambda)$ increasing in λ. The gain from manipulation is large when signals are intrinsically relatively precise and there is a high preference for coordination.

The key to this result is that receivers' respond less to their individual signals when information is manipulated than when it is not, i.e., $k^* < k^*_{\text{MS}}$. Intuitively, this is because with manipulation the signals x_i are endogenously less precise than in the Morris-Shin benchmark and hence each individual's actions are less responsive to their x_i.

Then knowing $k^* < k^*_{\text{MS}}$ and that $v_{\text{MS}}(k)$ is strictly decreasing in k, we then have the inequality (a) $v_{\text{MS}}(k^*) > v_{\text{MS}}(k^*_{\text{MS}})$. And since $1 - k^2 < 1$, we know $v(k) = (1-k)/(1+k) > (1-k)^2 = v_{\text{MS}}(k)$ for any k. Hence we also have the inequality (b) $v(k^*) > v_{\text{MS}}(k^*)$. Putting inequalities (a) and (b) together then implies that $V^*(\omega, \lambda) > V^*_{\text{MS}}(\omega, \lambda)$ for any $\omega \in (0, 1)$ and $\lambda \in (0, 1)$. In short, the sender is *always* better off with information manipulation.

Since the sender's payoffs depend on ω, λ only via the coefficient k^* we can then use Proposition 3 to calculate the comparative statics of V^* and V^*_{MS} with respect to ω, λ. For example, consider the effects of changing the intrinsic relative precision ω. If $\omega = 0$, then, in either scenario, information receivers will not respond to their signal, i.e., $k^* = k^*_{\text{MS}} = 0$ and the sender in either scenario will have (normalized) payoffs $V^* = V^*_{\text{MS}} = 1$ independent of λ. But as ω increases, the coefficients k^* and k^*_{MS} both increase so that the sender's payoffs V^* and V^*_{MS} both decrease with $V^* > V^*_{\text{MS}}$ for all ω and λ, as illustrated in the left panel of Figure 5. Moreover as $\omega \to 1$ we have that without manipulation $k^*_{\text{MS}} \to 1$ so that $V^*_{\text{MS}} \to 0$ but with manipulation $k^* \to \underline{k}(\lambda) < 1$ so that in the limit we have a strictly positive difference with limit

$$\underline{V}^*(\lambda) = \frac{1 - \underline{k}(\lambda)}{1 + \underline{k}(\lambda)} > 0 \tag{37}$$

that is strictly increasing in λ since $\underline{k}(\lambda)$ is strictly decreasing in λ.

While both V^* and V^*_{MS} are both strictly decreasing in ω, the sender's *gain from manipulation* $V^* - V^*_{\text{MS}}$ is non-monotonic in ω. In particular, for low ω the sender's payoffs with manipulation decrease more slowly in ω than do the benchmark Morris-Shin payoffs so that the sender's gain from manipulation is increasing for low ω. More precisely, for each λ there is a critical point $\omega^*(\lambda)$ such that the gain from manipulation is increasing in ω for all $\omega < \omega^*(\lambda)$ and then decreasing in ω for all $\omega > \omega^*(\lambda)$, as shown in the right panel of Figure 5. In short, the sender's gain from manipulation is large when signals are intrinsically relatively precise and there is a high preference for coordination. Of course, overall the sender would always prefer the intrinsic relative precision be low; indeed when ω is very low, there is negligible additional gain from being able to manipulate precisely because the sender's payoffs are already as high as they can be.

Why is the sender's gain from information manipulation large when signals are intrinsically relatively precise and there is a high preference for coordination? In short, the sender's gain is large when (i) *in the absence of manipulation* individuals are highly responsive to their individual signals, leading the aggregate A to be tightly clustered around θ, but where also (ii) *in the presence of manipulation* individuals are not responsive to their individual signals, leading the aggregate A to be more highly dispersed around θ. Now in general (ii) can be true either because coordination motives are strong or because individual signals are intrinsically relatively imprecise, but (i) can only be true if individual signals are intrinsically highly precise. Hence (i) and (ii) can only both be true at the same time if signals are intrinsically highly precise, say $\omega \to 1$ and coordination motives are strong, say $\lambda \to 1$. Put differently, the sender's gain is large when k^* is substantially less than k^*_{MS} so that with manipulation receivers respond much more to the common z. And from Proposition 3 we know that k^* is substantially less than k^*_{MS} when ω is high and λ is high.

Information manipulation and social coordination. Put differently, there is an interaction between the information manipulation effect and the social coordination effect from the benchmark Morris-Shin setting that can work to give the sender a large gain from manipulation. To see this, recall that in the Morris-Shin benchmark as $\omega \to 1$ we have $k^*_{\text{MS}} \to 1$ for all $\lambda \in (0, 1)$ — that is, even when there is a very powerful coordination motive, say $\lambda = 0.9$, then if signals are sufficiently precise relative to the prior then individual actions will still respond heavily to their signal despite the strong preference for coordination. In the Morris-Shin benchmark, this is a terrible outcome for the sender because it means that the receivers are perfectly coordinated on θ. With information manipulation, every receiver knows their signals are less informative than their intrinsic precision warrants, and this makes them respond less to their signal x_i (and more to the common z) than they otherwise would. This would be true even if $\lambda = 0$ — this is why in Proposition 3 we have $\lim_{\lambda \to 0} k^* := \overline{k}(\omega) < \omega$

for all ω — but this effect is larger when there is a coordination motive, $\lambda > 0$, because then it is not just that each individual responds more to z but also that each individual knows that each individual is responding more to z. In the presence of this "coordination multiplier", individual receivers are, in equilibrium, substantially less responsive to their x_i and more responsive to z and thus the aggregate A is more dispersed around the true θ, thus increasing the sender's payoffs. For example, when $\omega \to 1$ and $\lambda = 0.9$, say, the Morris-Shin payoffs are low $V^*_{\text{MS}} \to 0$ and the sender has high gains from manipulation, as shown in the right panel of Figure 5.

Given that the sender unambiguously benefits from manipulation, it is perhaps not surprising that the receivers are made unambiguously worse off.

Receivers made worse off. I measure the receivers' welfare in terms of their ex ante expected loss

$$\mathcal{L} = \lambda\mathbb{E}[(a(x_i, z) - A(y(\theta, z), z))^2] + (1-\lambda)\mathbb{E}[(a(x_i, z) - \theta)^2] \tag{38}$$

In the Morris-Shin benchmark with no information manipulation we have $y = \theta$ and since $a(x_i, z) = kx_i + (1-k)z$ and $A(\theta, z) = k\theta + (1-k)z$ we can collect terms and simplify to obtain the loss function

$$\mathcal{L}_{\text{MS}}(k\,;\,\omega, \lambda) := \frac{k^2}{\omega\Delta} + (1-\lambda)\frac{(1-k)^2}{(1-\omega)\Delta}, \qquad \Delta := \alpha + \beta \tag{39}$$

where $\Delta := \alpha+\beta$ is the *total* intrinsic precision (i.e., the prior precision plus the precision that signals would have if there was no manipulation; equivalently $\alpha = \omega\Delta$ and $\beta = (1-\omega)\Delta$). Evaluating this at $k = k^*_{\text{MS}}(\omega, \lambda)$ from (30) then gives the equilibrium loss in the case of no information manipulation, which I denote by $\mathcal{L}^*_{\text{MS}}(\omega, \lambda) := \mathcal{L}_{\text{MS}}(k^*_{\text{MS}}(\omega, \lambda); \omega, \lambda)$.

Similarly, *with* information manipulation we have $a(x_i, z) = kx_i + (1-k)z$ and $A(y, z) = ky+(1-k)z$ and now $y(\theta, z) = (k\theta+z)/(1+k)$ so that again collecting terms and simplifying from (38) we get the loss function

$$\mathcal{L}(k\,;\,\omega, \lambda) = \frac{k^2}{\omega\Delta} + (1-\lambda)\frac{(1+k)^{-2}}{(1-\omega)\Delta} \tag{40}$$

and evaluating this at the $k = k^*(\omega, \lambda)$ implied by (27) then gives the equilibrium loss when the sender can manipulate information, which I denote by $\mathcal{L}^*(\omega, \lambda) := \mathcal{L}(k^*(\omega, \lambda); \omega, \lambda)$.

We then have:

Proposition 6.

(i) Information manipulation is always payoff-reducing for receivers, $\mathcal{L}^* > \mathcal{L}^*_{\text{MS}}$.

(ii) Both $\mathcal{L}^*$ and $\mathcal{L}^*_{\text{MS}}$ are (a) strictly increasing in ω for all $\lambda \in (0, 1)$ and (b) and strictly decreasing in λ for all $\omega \in (0, 1)$.

(iii) The receivers' expected **loss from manipulation**, $\mathcal{L}^* - \mathcal{L}^*_{\text{MS}}$, is also (a) strictly increasing in ω for all $\lambda \in (0, 1)$ and (b) and strictly decreasing in λ for all $\omega \in (0, 1)$.

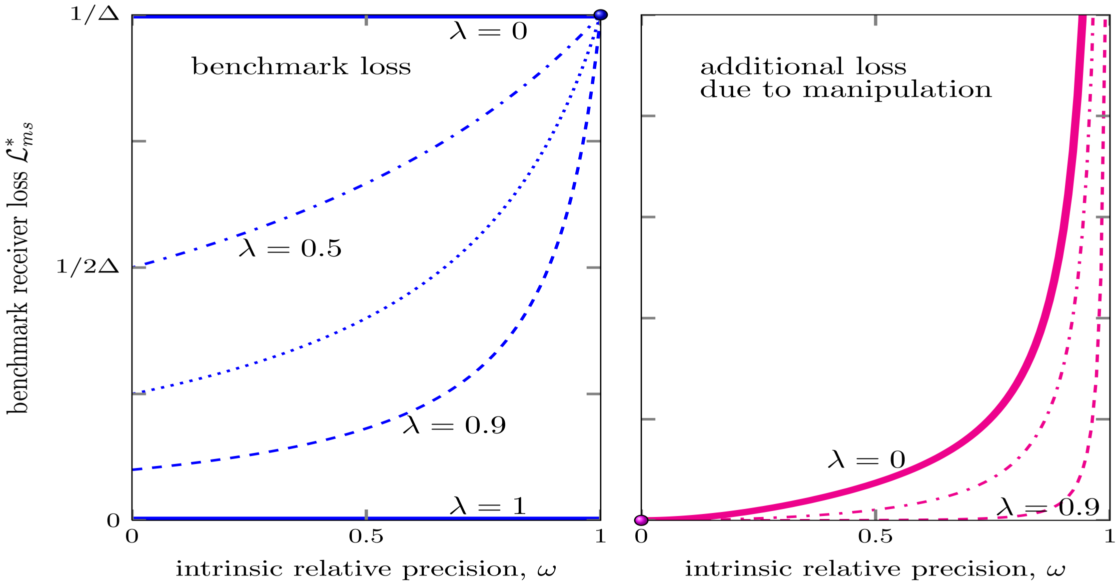


Figure 6: Receivers made worse off by manipulation.

The left panel shows the benchmark receiver loss when there is no information manipulation, $\mathcal{L}^*_{\text{MS}}(\omega, \lambda)$, as a function of the intrinsic relative precision ω and the preference for coordination λ. The benchmark loss is increasing in ω and decreasing in λ. The right panel shows the additional loss $\mathcal{L}^*(\omega, \lambda) - \mathcal{L}^*_{\text{MS}}(\omega, \lambda)$ due to manipulation, which is also increasing in ω and decreasing in λ. The receivers lose most from manipulation when they are intrinsically inclined to rely on their signals, i.e., when ω is high and λ is low, and yet those signals are contaminated by the sender's manipulation.

This result is illustrated in Figure 6. The left panel shows the Morris-Shin benchmark receiver loss $\mathcal{L}^*_{\text{MS}}$ as a function of ω for various λ. The loss is strictly increasing in ω for each $\lambda \in (0, 1)$ and strictly decreasing in λ for each $\omega \in (0, 1)$. In the special case of no coordination motive, $\lambda \to 0$, we have $k^*_{\text{MS}} = \omega$ and hence, from (39), the loss collapses to $\mathcal{L}^*_{\text{MS}} = \omega/\Delta + (1-\omega)/\Delta = 1/\Delta$ independent of ω. In the special case that receivers care only about coordination, $\lambda \to 1$, we have $k^*_{\text{MS}} = 0$ (i.e., actions load completely on the common z) and hence the loss collapses to $\mathcal{L}^*_{\text{MS}} = 0$ independent of ω. The right panel of Figure 6 shows the additional *loss from manipulation*, $\mathcal{L}^* - \mathcal{L}^*_{\text{MS}}$ as a function of ω for various λ. The loss from manipulation is likewise strictly increasing in ω for each $\lambda \in (0, 1)$ and strictly decreasing in λ for each $\omega \in (0, 1)$. Since the *level* of the loss with manipulation $\mathcal{L}^*$ is the sum of the left and right panels, it too is strictly increasing in ω and strictly decreasing in λ.

To summarize, the receivers are always made worse off by the sender's manipulation and the extent of their losses are greatest when the intrinsic relative precision of their signals is high or when there is a weak preference for coordination (i.e., when ω is high and λ is low). In other words, the receivers lose most when the fundamentals of the economy are such that they are intrinsically inclined to respond heavily to their signals and yet those signals are contaminated by the sender's manipulation. Notice, however, that for any fixed ω a higher λ reduces the receivers' loss from manipulation. This is because with high λ receivers care mostly about aligning their individual action a_i with the aggregate action A and hence the presence of information manipulation, which makes them less responsive to their individual

x_i and more responsive to their common z, will reduce the dispersion of their a_i around A and make the receivers marginally better off. To be clear, the receivers' losses from manipulation are extremely high when $\omega \to 1$, it is just that for any fixed ω they are marginally better off if λ is lower.

7 Conclusions

This paper studies information manipulation in a sender/receiver game with many imperfectly coordinated information receivers. The preferences of individual receivers balance a desire to match their individual action a_i to an unknown state of the world θ with their desire to match their action to the aggregate action of the population A. The information sender is informed about θ and seeks to *prevent* the receivers coordinating on it. In an effort to prevent coordination, the sender takes a costly hidden action that influences the informativeness of the receivers' signals on θ.

This game has a unique *linear* equilibrium. In that equilibrium, the sender is unable to introduce any bias into the receivers' information. Despite this lack of bias, the information sender always gains from the ability to manipulate information. This is because the sender's manipulation endogenously decreases the precision of individual signals making it harder to coordinate on θ. The sender's gain from information manipulation is large when the fundamentals of the economy are such that (i) *in the absence of manipulation* receivers would be highly responsive to their individual x_i, leading the aggregate A to be tightly clustered around θ, but where also (ii) *in the presence of manipulation* individuals are not responsive to their individual signals, leading the aggregate A to be more highly dispersed around θ. While (ii) could be true either because coordination motives are strong or because individual signals are intrinsically imprecise, leading receivers to rely more on their prior, (i) can only be true if individual signals are intrinsically precise. Hence (i) and (ii) can only both be true at the same time if signals are intrinsically precise and coordination motives are strong. If the fundamentals of the economy are such that (i) and (ii) are both true, then the sender's information manipulation technology makes receivers much less responsive to their signals, which increases the dispersion of A around θ and gives the sender a large increase in payoffs.

Given that the information manipulation technology is always payoff-improving for the sender, it is not too surprising that the manipulation also always makes the receivers worse off. The receivers' losses from information manipulation are large when signals are intrinsically relative precise — i.e., when the receivers are fundamentally inclined to respond heavily to their signals but refrain from doing so given the sender's manipulation. Finally, while a strong coordination motive gives the sender large gains from manipulation, for the receivers a strong coordination motive to some extent *mitigates* their losses. This is because when receivers have a strong coordination motive, they mostly care about aligning their individual

a_i with the aggregate A. Thus the information manipulation, which makes receivers respond more to the common z and less to their individual x_i, thereby reducing the dispersion of their a_i around A, has a smaller welfare cost to the receivers than it would have if the receivers had weak coordination motives.

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